Broadband Linear Silicon Mach-Zehnder Modulators

Cheryl Sorace, Anatol Khilo, Franz X. Kärtner
Department of Electrical Engineering and Computer Science, and Research Laboratory of Electronics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA
e-mail: ch20117@mit.edu

Abstract: We show that properly dimensioned push-pull Mach-Zehnder modulators using reverse biased silicon diodes exhibit superior linearity (>60dB) over conventional Lithium Niobate Mach-Zehnder modulators, making them attractive for analog electronic to photonic conversion systems.

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1. Introduction

Over the last decade dramatic progress has been made in the development of silicon based light modulators and silicon photonics in general [1-3]. Most of these efforts have been focused on lowering the drive voltage and power–mainly for optical communications purposes. Linearized modulators are important components of analog RF-photonic systems and various techniques have been used in the past to linearize the Mach-Zehnder (MZ) characteristic of conventional Lithium-Niobate modulators [4]. Usually these techniques are based on using multiple MZ modulators to achieve overall linearity, which increases the system complexity.

In this paper, we consider a push-pull MZ modulator with reverse-biased Si diodes in both arms (see Fig. 1). The phase response of reverse-biased diodes is a nonlinear function of applied voltage. In addition, free-carrier induced loss is present and is a nonlinear function of voltage as well. Because of these nonlinearities, one might expect the linearity of the MZ modulator to be rather poor. However, we show that by proper design the quasi-static nonlinearities of a Si MZ modulator can be greatly suppressed (-60dBc or more), showing performance superior to that of Lithium-Niobate modulators. Such devices are of great importance for the development of silicon photonic based analog systems, such as photonic analog-to-digital converters (ADCs) [5].

2. Analysis of Modulator Linearity

Consider a Mach-Zehnder modulator with two identical, non-ideal phase shifting sections placed into its arms, driven in a push-pull configuration (see Fig. 1). The phase shifters are described by a voltage-dependent phase $\varphi(v)$ and an absorption coefficient $\alpha(v)$. A $\pi/2$ phase shift between the arms is used to place the MZ-modulator in quadrature. The power at the top and bottom outputs of a MZ modulator with ideal 50/50 input and output couplers can be expressed as

$$P_{top} = P_0 \exp\left[-\left(\frac{\alpha_1}{2}+\frac{\alpha_2}{2}\right)L\right]\sin^2\left(\frac{\pi}{4}+\frac{\varphi_1-\varphi_2}{2}\right)+\frac{P_0}{4}\left[\exp\left(-\frac{\alpha_1 L}{2}\right)-\exp\left(-\frac{\alpha_2 L}{2}\right)\right]^2, \quad (1)$$

$$P_{bottom} = P_0 \exp\left[-\left(\frac{\alpha_1}{2}+\frac{\alpha_2}{2}\right)L\right]\cos^2\left(\frac{\pi}{4}+\frac{\varphi_1-\varphi_2}{2}\right)+\frac{P_0}{4}\left[\exp\left(-\frac{\alpha_1 L}{2}\right)-\exp\left(-\frac{\alpha_2 L}{2}\right)\right]^2, \quad (2)$$

where $\varphi_1 = \varphi(v_{DC}+v)$ and $\varphi_2 = \varphi(v_{DC}-v)$ are the phase shifts in the top and bottom arms, $\alpha_1 = \alpha(v_{DC}+v)$ and $\alpha_2 = \alpha(v_{DC}-v)$ are the power loss coefficients in the top and the bottom arms, and P_0 is the input power.
Expanding $\varphi(v)$ and $\alpha(v)$ into Taylor series around the DC voltage, v_{DC}, and keeping the first three terms, we get

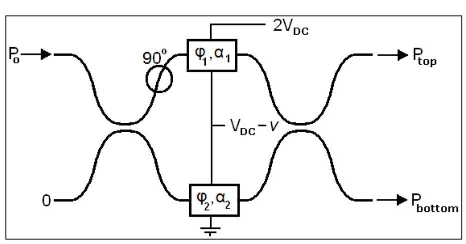

Fig. 1. MZ modulator configuration.

$$\varphi(v) = \left(\varphi_{DC} + av + \frac{bv^2}{2} + \frac{cv^3}{6} \right) L, \qquad \alpha(v) = \left(\alpha_{DC} + xv + \frac{yv^2}{2} + \frac{zv^3}{6} \right) L, \tag{3}$$

where L is the length of the phase shifting section. Plugging (3) into (1) and (2), and expanding the obtained expression in a Taylor series we finally arrive at

$$P_{top} = P_0 e^{-\alpha_{DC} L} \left[\frac{1}{2} + (aL)v + \frac{1}{4}\left(x^2L^2 - yL\right)v^2 + \left(-\frac{2}{3}a^3L^3 + \frac{c}{6}L - \frac{1}{2}ayL^2 \right) v^3 \right] \tag{4}$$

$$P_{bottom} = P_0 e^{-\alpha_{DC} L} \left[\frac{1}{2} - (aL)v + \frac{1}{4}\left(x^2L^2 - yL\right)v^2 - \left(-\frac{2}{3}a^3L^3 + \frac{c}{2}L - \frac{1}{2}ayL^2 \right) v^3 \right] \tag{5}$$

These equations suggest that the MZ modulator outputs contain undesirable quadratic and cubic terms. For linear operation, these terms should be made as small as possible. Let us analyze these terms in more detail.

According to (4)-(5), the third order term is a sum of three components: (i) the cubic term of the sinusoidal MZ transfer function; present even if the phase shift is a perfectly linear function of voltage, (ii) the cubic term of the nonlinear phase $\varphi(v)$, and (iii) the quadratic term of the loss $\alpha(v)$. If the coefficients a, c, and y, have the appropriate signs, it is possible to find the length L at which the cubic nonlinearity cancels out completely. Note that such cancellation cannot be achieved in the case of a modulator with purely linear $\varphi(v)$ and $\alpha(v)$, such as a Lithium Niobate modulator.

The quadratic term of the modulator transfer function originates from the voltage-dependent loss. The quadratic nonlinearity is of concern for broadband analog systems such as photonic analog-to-digital converters. One way to remove it is to use both the lower and upper output ports as one differential output signal that feeds a balanced detector. The detected signal is then ~ $P_{top}-P_{bottom}$, which cancels the 2nd order nonlinearity according to Eqs. (4)-(5). Another way to remove the quadratic term is to move the modulator operation point away from the quadrature point. This introduces an additional quadratic term into the modulator transfer function, which will cancel the loss-induced quadratic nonlinearity in one modulator output.

Note that the quadratic term of the phase response $\varphi(v)$, which is often the largest nonlinear term of $\varphi(v)$, is automatically cancelled by the symmetry of the push-pull configuration.

3. Linear Operation of Reversely Biased Silicon MZ Modulator

The analysis of the previous section is general and can be applied to any MZ modulator with non-ideal phase shifters. We now consider a reverse-biased silicon diode with the cross-section shown in Fig. 2 to illustrate how the principles described above can be used to achieve linear modulation in practice. The diode cross-section, shown in Fig. 2, was not designed with linearization in mind.

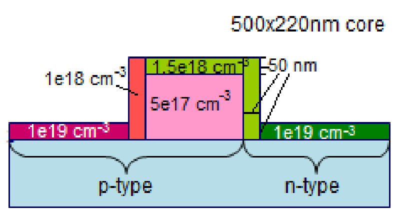

Fig 2. Cross-section of simulated silicon diode phase modulator to be operated in reverse bias

Fig. 3a shows the DC phase change $\varphi(v)$ and absorption $\alpha(v)$ caused by the diode as a function of reverse bias voltage. Electrical simulations were carried out in Sentaurus™ and optical ones in MATLAB. A wavelength of 1550nm was assumed. Both $\varphi(v)$ and $\alpha(v)$ are highly non-linear functions of voltage, having roughly a square-root dependence. The square root expansion coefficients a and c have the same sign and thus cancellation of the cubic nonlinearity is possible at some L.

Fig. 3b shows the optimal length L, determined numerically by applying a sinusoidal RF tone to the modulator input, calculating the output based on the responses of Fig. 3a, and finding the length which minimizes the 3rd harmonic at the output. Because the nonlinearity varies as a function of bias voltage, the optimal length also varies. Therefore, the bias point can be tuned to correct for small errors in simulation or fabrication.

Note, that the lengths minimizing the 3rd harmonic are reasonable for a silicon MZ modulator; indeed, modulators of these lengths have been successfully fabricated and demonstrated in the past (see, for example, [3]). They are short enough that losses are tolerable and the footprint is manageable, yet long enough to allow for a

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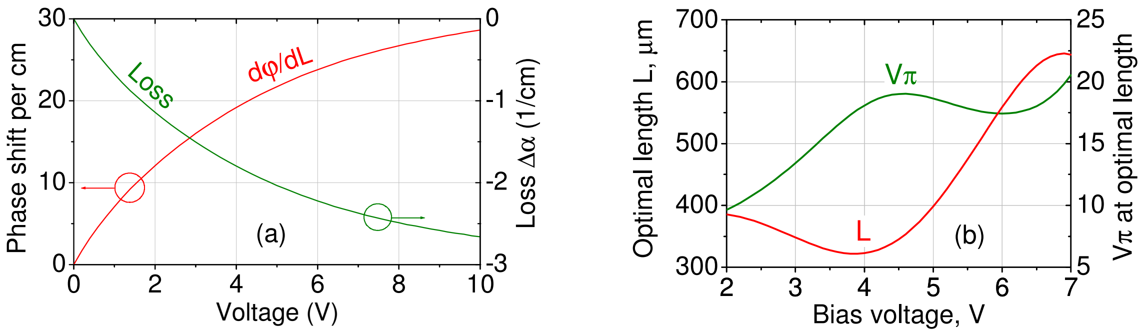

Fig. 3. (a) Phase and absorption responses of the modulator with applied DC voltage. (b) Length which minimizes the third harmonic for a given bias voltage, and associated small-signal V_π at this length and bias voltage.

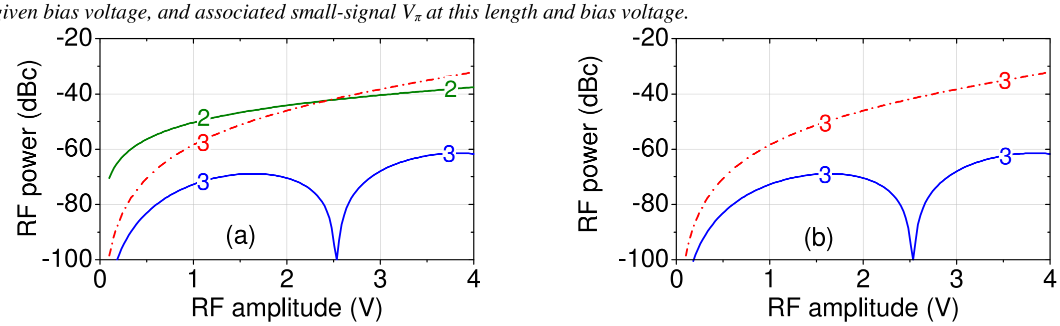

Fig. 4. (a) 2nd and 3rd harmonics of the optimized 365µm-long Si diode MZ-modulator with 5V bias voltage (solid lines); (b) the same with 2nd harmonic eliminated. For reference, the 3rd harmonic of an MZ modulator with ideally linear phase shifters is included (dashed red line).

reasonable modulation depth. To illustrate this last point, the small-signal V_π associated with each length is plotted in Fig. 3b.

Fig. 4 illustrates the suppression of the third-order nonlinearity in a silicon MZ modulator with L=365µm at a 5V bias voltage. Fig. 4(a) shows the power of the 2nd and 3rd harmonics calculated for a sinusoidal driving signal input as a function of its amplitude. For comparison, Fig. 4(a) also shows the 3rd harmonic for an "ideal" MZ modulator, i.e. a modulator with perfectly linear phase shifters. The 3rd harmonic of the optimized Si MZ modulator is 10-30dB below that of the "ideal" MZ modulator. The remaining 3rd harmonic content is due to higher order nonlinear terms in the Taylor expansion, such as the fifth order term, contributing to the 3rd harmonic. The 2nd harmonic, caused by the voltage-dependent absorption, is plotted for completeness in Fig. 4(a). As explained above, the 2nd harmonic can be completely removed either by using a differential detection scheme, or by tuning the MZ operating point (Fig. 4b).

Other important considerations are the wavelength dependence of the nonlinearity cancelation and the behavior at high frequencies, which will presented at the conference.

4. Conclusion

We have shown that the nonlinearity of the phase shifter sections of a MZ modulator can be used to cancel the nonlinearity of the sinusoidal MZ transfer function to obtain highly linear operation. To achieve high linearity, the modulator must be driven in a push-pull configuration and the quadratic term must be eliminated as described above. In the case of a reversed-biased silicon MZ-modulator, the 2nd and 3rd harmonics can be suppressed to below -60dBc, well below the level of a MZ-modulator with linear phase shifters. This can be achieved for a modulator with reasonable length and sensitivity. These linearized modulators can be useful in analog applications; for example in photonic analog-to-digital converters they would allow a resolution of up to 10 effective bits.

5. References

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